

Alcuni esercizi sugli integrali

N.B. I risultati NON sono necessariamente INTERI

Integrali immediati

- $\int_0^1 e^{-x} dx =$
- $\int_0^1 e^x dx =$
- $\int_0^1 sh(x) dx =$
- $\int_0^1 ch(x) dx =$
- $\int_0^1 3x^2 dx =$
- $\int_0^1 \frac{1}{1+x^2} dx =$
- $\int_0^{\pi/4} \frac{1}{\sqrt{1-x^2}} dx =$
- $\int_0^{\pi/4} \frac{1}{\cos^2(x)} dx =$
- $\int_1^2 \frac{1}{x} dx =$
- $\int_0^{\pi} \sin(x) dx =$
- $\int_0^{\pi/2} \cos(x) dx =$

Integrali quasi immediati

- $\int_0^1 (-2)e^{-2x} dx =$
- $\int_0^1 3e^{3x} dx =$
- $\int_0^1 5 sh(5x) dx =$
- $\int_0^1 (-3)ch(-3x) dx =$
- $\int_0^1 \frac{2}{1+4x^2} dx =$
- $\int_0^{\pi/8} \frac{2}{\sqrt{1-4x^2}} dx =$
- $\int_0^{\pi/8} \frac{2}{\cos^2(2x)} dx =$
- $\int_1^2 \frac{3}{1+3x} dx =$

- $\int_0^\pi 5 \sin(5x) dx =$
- $\int_0^{\pi/6} 3 \cos(3x) dx =$
- $\int_0^1 (-2x) e^{-x^2} dx =$
- $\int_0^1 (3x^2) e^{x^3} dx =$
- $\int_0^1 (5x^4) \operatorname{sh}(x^5) dx =$
- $\int_0^1 (-3x^2) \operatorname{ch}(-x^3) dx =$
- $\int_0^1 \frac{2x}{1+x^4} dx =$
- $\int_0^{\pi/8} \frac{2x}{\sqrt{1-x^4}} dx =$
- $\int_0^{\pi/8} \frac{2x}{\cos^2(x^2)} dx =$
- $\int_1^2 \frac{3x^2}{1+x^3} dx =$
- $\int_0^\pi 5x^4 \sin(x^5) dx =$
- $\int_0^{\pi/6} 3x^2 \cos(x^3) dx =$

Integrali molto facili

- $\int_0^1 x e^{-x^2} dx =$
- $\int_0^1 x^2 e^{x^3} dx =$
- $\int_0^1 x^4 \operatorname{sh}(x^5) dx =$
- $\int_0^1 x^3 \operatorname{ch}(x^4) dx =$
- $\int_0^1 \frac{x}{1+x^4} dx =$
- $\int_0^{\pi/8} \frac{1}{\sqrt{1-4x^2}} dx =$
- $\int_0^{\pi/8} \frac{1}{\cos^2(2x)} dx =$
- $\int_1^2 \frac{x}{1+3x^2} dx =$
- $\int_0^\pi x \sin(5x^x) dx =$
- $\int_0^{\pi/6} (-2x) \cos(3x^2) dx =$

Integrali per parti

- $\int_0^1 x e^{-x} dx =$
- $\int_0^1 x e^{2x} dx =$
- $\int_0^1 x \operatorname{sh}(x) dx =$
- $\int_0^1 \log(x) dx =$
- $\int_0^1 \operatorname{arctg}(x) dx =$
- $\int_0^\pi x \sin(x) dx =$
- $\int_0^\pi (-2x) \cos(3x) dx =$

*Verificare (e imparare a memoria) i seguenti
integrali notevoli*

- $\int_0^{\pi/2} \sin^2(x) dx = \pi/4$
- $\int_0^{\pi/2} \cos^2(x) dx = \pi/4$
- $\int_{\pi/2}^\pi \sin^2(x) dx = \pi/4$
- $\int_{\pi/2}^\pi \cos^2(x) dx = \pi/4$
- $\int_\pi^{3\pi/2} \sin^2(x) dx = \pi/4$
- $\int_\pi^{3\pi/2} \cos^2(x) dx = \pi/4$
- $\int_{3\pi/2}^{2\pi} \sin^2(x) dx = \pi/4$
- $\int_{3\pi/2}^{2\pi} \cos^2(x) dx = \pi/4$
- $\int_0^1 x^k dx = \frac{1}{k+1} \quad \text{per } k > -1$
- $\int_0^{+\infty} x^0 e^{-x} dx \equiv \int_0^{+\infty} e^{-x} dx = 1$
- $\int_0^{+\infty} x^1 e^{-x} dx \equiv \int_0^{+\infty} x e^{-x} dx = 1$
- $\int_0^{+\infty} x^2 e^{-x} dx = 2$
- $\int_0^{+\infty} x^3 e^{-x} dx = 2 \cdot 3$
- $\int_0^{+\infty} x^4 e^{-x} dx = 2 \cdot 3 \cdot 4$
- $\int_0^{+\infty} x^n e^{-x} dx = n! \quad \text{per } n \text{ intero } \geq 0$

Integrali di funzioni definite a pezzi

- Sia $f(x)$ data da

$$f(x) = \begin{cases} 1 & \text{per } x \leq 0 \\ e^x & \text{per } 0 < x \leq \log 2 \\ 2 & \text{per } \log 2 < x \end{cases}$$

Disegnare la funzione e calcolare $\int_{-2}^4 f(x) dx$

- Sia $f(x)$ data da

$$f(x) = \begin{cases} \sin(x) & \text{per } 0 \leq x \leq \pi/2 \\ 1 & \text{per } \pi/2 < x \leq 4 \\ x - 3 & \text{per } 4 < x \end{cases}$$

Disegnare la funzione e calcolare $\int_0^6 f(x) dx$

- Sia $f(x)$ data da

$$f(x) = \begin{cases} x + 1 & \text{per } -1 \leq x \leq 0 \\ x - 1 & \text{per } 0 < x \leq 1 \\ 1 & \text{per } 1 < x \end{cases}$$

Disegnare la funzione e calcolare $\int_{-1}^3 f(x) dx$

- Sia $f(x)$ data da

$$f(x) = \begin{cases} (x + 1)^2 & \text{per } -1 \leq x \leq 0 \\ 1 - x^2 & \text{per } 0 < x \leq 1 \\ 0 & \text{per } 1 < x < 2 \\ x - 2 & \text{per } 2 \leq x \end{cases}$$

Disegnare la funzione e calcolare $\int_{-1}^4 f(x) dx$

- Sia $f(x)$ data da

$$f(x) = \begin{cases} -1 & \text{per } x \leq -\pi \\ \sin(x^3) & \text{per } -\pi < x \leq \pi \\ 1 & \text{per } \pi < x < 4 \\ 2 & \text{per } 4 \leq x \end{cases}$$

Disegnare la funzione e calcolare $\int_{-4}^6 f(x) dx$